DCT-003-1013008

DCT-003-1013008

Seat No. ______

B. Sc. (Sem. III) Examination

August - 2022

Mathematics : Paper-3(A)

(Real Analysis)

Faculty Code : 003

Subject Code : 1013008

Time : **2:30** Hours] [Total Marks : **70**

Instructions: (1) Attempt any Five questions out of Ten questions.

(2) Figure to the right indicate full marks of the questions.

1 (A) Answers the following questions. **4**

(1) Define: Monotonically Increasing Sequence

(2) Define: Limit point of a Sequence

(3) $\lim\limits_{n\to\infty} \frac{1}{n^2} = \underline{\quad ? \quad}$

(4) True or False: The sequence $\{(-1)^n\}$ is not convergent.

(B) Attempt the following. **2**

(1) Show that $\lim\limits_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]=0$.

(C) Attempt the following. **3**

(1) Show that every convergent sequence is bounded.

(D) Attempt the following. **5**

Prove that: The limit of a convergent sequence is unique.

2 (A) Answer the following questions. **4**

(1) Define: Bounded Sequence

(2) Define: Sequence

(3) $\lim_{n\to\infty} \frac{1}{2n+1} = \underline{\quad ? \quad}$

(4) True of False: The sequence $\{(-1)^{2n}\}$ is convergent.

(B) Attempt the following: **2**

(1) Find limit points of the sequence $\{(-1)^n\}$.

(C) Attempt the following. **3**

(1) Show that: $\lim_{n\to\infty} \frac{(3n+1)(n-2)}{n(n+3)} = 3$.

(D) Attempt the following. **5**

(1) Using definition show that: $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

3 (A) Answer the following questions. **4**

(1) Define: Sequence of partial sum

(2) Define: Positive term series

(3) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \underline{\quad ? \quad}$

(4) Decide the series $\sum_{n=1}^{\infty} \frac{1}{n^e}$ is convergent or divergent?

(B) Attempt the following: **2**

(1) State Logarithmic Test.

(C) Attempt the following. **3**

(1) Show that: If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

(D) Attempt the following. **5**

(1) Show that the series $\frac{1\,.\,2}{3^2\,.\,4^2} + \frac{3\,.\,4}{5^2\,.\,6^2} + \frac{5\,.\,6}{7^2\,.\,8^2} + \cdots\cdots$ is convergent.

4 (A) Answer the following questions. **4**

(1) Define: Alternate Series

(2) Define: Convergence of a Series

(3) $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n = $ ___?___

(4) Decide the series $\sum_{n=1}^{\infty}\frac{1}{n^{\pi}}$ is convergent or divergent?

(B) Attempt the following: **2**

(1) State Rabbe's Test.

(C) Attempt the following. **3**

(1) Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+.....$ is not convergent.

(D) Attempt the following. **5**

(1) Test for the convergence of the series

$\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+1}x^n,\ x>0$.

5 (A) Answer the following questions. **4**

(1) Define: Curl of a vector point function.

(2) Define: Divergence of a vector point function.

(3) If $\overline{A}=x^2\hat{i}+xy\hat{j}+yz\hat{k}$, then find $\overline{A}\times\overline{A}$.

(4) True or False: Curl is a vector quantity.

(B) Attempt the following: **2**

(1) If $\phi=3x^2y-yz^2$; find grad ϕ at the point (1, –2, –1).

(C) Attempt the following. **3**

(1) If $\overline{r}=x\hat{i}+y\hat{j}+z\hat{k}$, show that grad $\left(\frac{1}{r}\right)=-\frac{\overline{r}}{r^3}$.

(D) Attempt the following. **5**

(1) Let $\overline{a}$ be a constant vector. Prove that.

(a) div $(\overline{r}\times\overline{a})=0$.

(b) curl $(\overline{r}\times\overline{a})=-2\overline{a}$.

6 (A) Answer the following questions. **4**

(1) Find the unit normal to the surface $z = 2xy$ at the point (2, 1, 4)

(2) Find curl (grad f), where $f = 2x^2 - 3y^2 + 4z^2$.

(3) Define: Gradient of a scalar point function.

(4) True or False: Gradient is a vector quantity.

(B) Attempt the following: **2**

(1) If $\overline{v} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$, find the value of div $\overline{v}$.

(C) Attempt the following. **3**

(1) Prove that: The function $H = e^{-\lambda x}(C_1 \sin \lambda y + C_2 \cos \lambda y)$ satisfy the Laplace equation. Where λ, C_1 and C_2 are arbitraty constants.

(D) Attempt the following. **5**

(1) Show that: div (grad r^n) = $n(n + 1)r^{n-2}$, where $r = \sqrt{x^2 + y^2 + z^2}$.

7 (A) Answer the following questions. **4**

(1) What are the limits of x and y in the integral $\int_0^1 \int_{\sqrt{y+3}}^2 f(x, y)dA$.

(2) Evaluate: $\int_0^1 \int_0^2 dx\, dy$.

(3) $\int_0^1 \int_0^x e^x dxdy = $ ___?___

(4) $\int_{-a}^a \int_0^x dydx = $ ___?___

(B) Attempt the following: **2**

(1) Let $x = r\cos\theta$, $y = r\sin\theta$. Then find the Jacobian $J = \dfrac{\partial(x, y)}{\partial(r, \theta)}$.

(C) Attempt the following. **3**

(1) Change the order of the intergration

$\int_1^4 \int_{\sqrt{y}}^2 f(x,y)dx\, dy$.

(D) Attempt the following. **5**

(1) Evaluate: $\iint_R e^{2x+3y} dx\, dy$ over the region bounded by $x = 0$, $y = 0$ and $x + y = 1$.

8 (A) Answer the following questions. **4**

(1) $\int_1^0 \int_0^1 (x+y)dxdy =$ ___?___

(2) Let R be $x^2 + y^2 \leq 1$. What is the value of $\iint_R dxdy$?

(3) Evaluate: $\int_0^1 \int_0^x dy\, dx$.

(4) True or False: If limits of both the variables are constants in a double integral then the region of the integration is a rectangle.

(B) Attempt the following: **2**

(1) Evaluate: $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^1 r^2 \sin\theta\, dr\, d\theta\, d\phi$.

(C) Attempt the following. **3**

(1) Evaluate: $\int_0^{\frac{\pi}{2}} \int_{a(1-\cos\theta)}^{a} r^2\, dr\, d\theta$.

(D) Attempt the following. **5**

(1) Evaluate: $\iint_R (x^2 + y^2)\, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ by changing into polar coordinates.

9 (A) Answer the following questions. **4**

(1) Define: Beta function.

(2) The value of $\overline{|(1/2)} = \underline{\quad ? \quad}$

(3) The value of $\overline{|(-1/2)} = \underline{\quad ? \quad}$

(4) The value of $\beta(4, 5)$ is $\underline{\quad ? \quad}$

(B) Attempt the following: **2**

(1) Show that $\beta(m, n) = \beta(n, m)$.

(C) Attempt the following. **3**

(1) Evaluate: $\int_0^\infty \sqrt{x}\, e^{-3\sqrt{x}} dx$.

(D) Attempt the following. **5**

(1) Prove that: $\beta(m, n) = \dfrac{\overline{|m}\,\overline{|n}}{\overline{|m+n}}$.

10 (A) Answer the following questions. **4**

(1) Define: Gamma function.

(2) $\overline{|(5/2)} = \underline{\quad ? \quad}$

(3) $\int_0^\infty e^{-x} x^3\, dx = \underline{\quad ? \quad}$

(4) Find the value of $\overline{|6}$.

(B) Attempt the following: **2**

(1) State Stoke's Theorem.

(C) Attempt the following. **3**

(1) Evaluate: $\int_0^1 x^4 (1-\sqrt{x})^5 \, dx$.

(D) Attempt the following. **5**

(1) Using Green's Theorem evaluate $\int_c (x^2 y \, dx + x^2 \, dy)$ where c is the boundary described counter clockwise of the triangle with vertices (0, 0), (1, 0), (1, 1).
